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# Free vibration analysis of armchair double-walled carbon nanotubes embedded in an elastic medium using nonlocal Euler-Bernoulli beam theory.

Z. Mohamed[1,2], B. Mokhtar [3], D. Tahar Hassaine[1,2], H. Lazreg [1,2], T. Abdelouahed [2]

[1]Ibn Khaldoune , genie civil , Tiaret, Algeria
[2]djillali liabesse, genie civil, Sidi Bel Abbés, Algeria
[3]Béchar , genie civil, Béchar, Algeria

Zidour.mohamed@yahoo.fr
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In the present work, nonlocal Euler-Bernoulli beam theory is used to investigate the free vibration response of armchair double-walled carbon nanotube (DWCNT) embedded in an elastic medium. It is noticed in the present study that the equivalent Young's modulus for armchair DWCNT is derived using an energy-equivalent model. The results indicate the dependence of nonlocal effects, the mode number and Winkler modulus parameter on the frequency of DWCNT.

Carbon nanotubes are tubular structures with nanometre diameter and micrometer length. Since the single-walled carbon nanotube (SWCNT) and multi-walled carbon nanotube (MWCNT) are found by Iijima [1]. Recently, considerable attention has been turned to the mechanical behavior of single and multi-walled carbon nanotubes embedded in polymer or metal matrix [2,3]. Vibration and buckling analyses [4-5] of CNTs have shown the employment of Winkler-type elastic foundation for modelling continuous surrounding elastic medium.

Although several studies on the vibration behavior of CNTs have been carried out based on nonlocal Euler-Bernoulli beam theory, no studies can be found for the vibration behavior of armchair CNTs embedded in an elastic medium. In this paper, nonlocal Euler-Bernoulli beam theory has been implemented to investigate the vibration response of armchair double-walled carbon nanotubes (DWCNTs) embedded in an elastic medium.

The general equation for transverse vibrations of an (DWCNTs) elastic beam under distributed transverse pressure with the surrounding elastic medium on the basis of nonlocal elasticity. [6,7]

$$\boldsymbol{p}_{12} = \boldsymbol{EI}_1 \frac{\partial^4 \boldsymbol{w}_1}{\partial \boldsymbol{x}^4} + \boldsymbol{\rho}\, \boldsymbol{A}_1 \frac{\partial^2 \boldsymbol{w}_1}{\partial \boldsymbol{t}^2} - (\boldsymbol{e}_0 \boldsymbol{a})^2 \left( \boldsymbol{\rho}\, \boldsymbol{A}_1 \frac{\partial^4 \boldsymbol{w}_1}{\partial \boldsymbol{x}^2 \partial \boldsymbol{t}^2} - \frac{\partial^2 \boldsymbol{p}_{12}}{\partial \boldsymbol{x}^2} \right) \tag{1}$$

$$\boldsymbol{f} - \boldsymbol{p}_{12} = \boldsymbol{EI}_2 \frac{\partial^4 \boldsymbol{w}_2}{\partial \boldsymbol{x}^4} + \boldsymbol{\rho}\, \boldsymbol{A}_2 \frac{\partial^2 \boldsymbol{w}_2}{\partial \boldsymbol{t}^2} - (\boldsymbol{e}_0 \boldsymbol{a})^2 \left( \boldsymbol{\rho}\, \boldsymbol{A}_2 \frac{\partial^4 \boldsymbol{w}_2}{\partial \boldsymbol{x}^2 \partial \boldsymbol{t}^2} - \frac{\partial^2 \boldsymbol{f}}{\partial \boldsymbol{x}^2} + \frac{\partial^2 \boldsymbol{p}_{12}}{\partial \boldsymbol{x}^2} \right) \tag{2}$$

The parameters used in calculations for the armchair (DWCNTs) are given as follows: the effective thickness of (CNTs) taken to be 0.258 nm [8], the mass density ρ = 2.3 g cm-3 [9] and layer distance h=0.34 nm [10].

Figure 1 show the effect of small scale parameter on the higher vibration response of embedded armchair (DWCNT) with elastic medium modeled as Winkler-type foundation. The aspect ratio $L/d$ is taken as 40. The Winkler modulus ratio parameter ($k_W/c$) values were taken in the range of 0 – 60. From the figure, it is observed that with increase in $e_0a$ values, the frequencies obtained by nonlocal Euler-Bernoulli theory become smaller compared to local model. Furthermore, it is seen that as the Winkler modulus ratio parameter increases the frequency ratio increases. This increasing trend is attributed to the stiffness of the elastic medium. Figure 2 it is observed that the nonlocal effects on vibration response are more significant for higher modes of vibration. This is interpreted from the fact that frequency ratio values for higher modes ($k = 3, 4$) are quite less than $k = 1$. This significance of nonlocal effects in higher modes is attributed to the influence of small wavelength for higher modes. For smaller wavelengths, the interactions between atoms are increasing and this leads to an increase in the nonlocal effects. Furthermore, as the Winkler modulus parameter increases, the frequency ratios increase for higher modes except for first mode of vibration. This implies that there is comparatively less effect of elastic medium on higher mode frequency of armchair (DWCNT).

This paper studies the vibration of armchair (DWCNTs) embedded in elastic medium based on the

Euler-Bernoulli beam theory. According to the study, the results showed the dependence of the vibration characteristics on the nonlocal parameter, mode number and surrounding elastic medium. this dependence is attributed to the influence of small wavelength for higher modes. For smaller wavelengths, interactions between atoms are increasing and this leads to an increase in the nonlocal effects. Furthermore, as the Winkler modulus parameter increases, the frequency ratios increase for higher modes.


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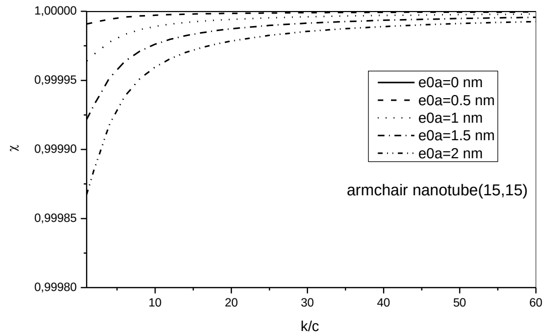


**Figure 1:** Effect of Winkler modulus ratio parameter on the higher frequency ratio of armchair DWCNT for various small-scale coefficients with ( $L/d = 40$ and $k = 6$ ).

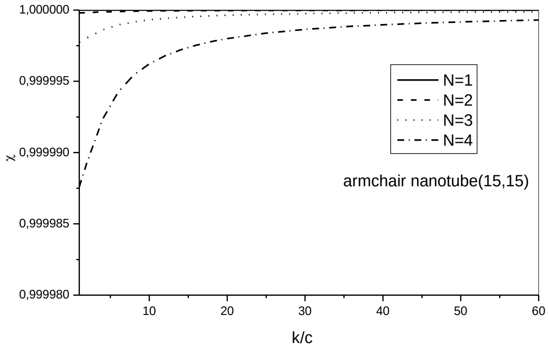


**Figure 2:** Effect of Winkler modulus ratio parameter on the higher frequency ratio of armchair DWCNT for various mode numbers with ( $e_0a = 2\,\mathrm{nm}$ and $L/d = 40$ ).