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In this article, thermal buckling stability analysis of moderately thick functionally graded rectangular and square plate is studied. The equilibrium and stability equations are derived using two variable refined plate theory.

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Keywords: Buckling; functionally graded plate; refined plate theory; thermal loading. The results are verified with the known data in the literature.

In this article, thermal buckling analysis of moderately thick functionally graded rectangular and square plate is studied. The equilibrium and stability equations are derived using two variable refined plate theory. The theory, which has strong similarity with classical plate theory in many aspects, accounts for a quadratic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. By using an analytical method, the stability equations are replaced by four equations. Solving the equations and satisfying the boundary conditions of simply supported edges, the critical buckling temperature is found analytically. Thermal buckling of functionally graded plate for three types of thermal loading, uniform temperature rise, linear temperature rise and non-linear temperature rise through the thickness are investigated. Finally, the effects of power law index and plate thickness on the critical buckling temperature of functionally graded rectangular and square plates are discussed in details.

The multilayered materials are used in many structures. In conventional laminated composite structures, homogeneous elastic laminae are bonded together to obtain enhanced mechanical and thermal properties. The main inconvenience of such an assembly is to create stress concentrations along the interfaces, more specifically when high temperatures are involved. This can lead to delaminations, matrix cracks, and other damage mechanisms which result from the abrupt change of the mechanical properties at the interface between the layers. One way to overcome this problem is to use functionally graded materials within which material properties vary continuously. The concept of functionally graded material (FGM) was proposed in 1984 by the material scientists in the Sendai area of Japan [1].

The FGM is a composite material whose composition varies according to the required performance. It can be produced with a continuously graded variation of the volume fractions of the constituents. That leads to a continuity of the material properties of FGM: this is the main difference between such a material and an usual composite material. The FGM is suitable for various applications, such as thermal coatings of barrier for ceramic engines, gas turbines, nuclear fusions, optical thin layers, biomaterial electronics, etc.

Many investigations have been carried out on the subject of mechanical and thermal buckling of structures. Developments of new materials such as functionally graded materials (FGMs), however, have necessitated more research in this area. The response of a functionally graded ceramic–metal plate was investigated by Praveen and Reddy using a finite element model that accounts for the transverse shear strains, rotary inertia, and moderately large rotations in the Von Karman sense [2]. In Ref. [3], Reddy et al. developed the relationship between the bending solutions of the classical plate theory and the first order plate theory for functionally graded circular plates. Bouzza et al reported mechanical and thermal buckling of rectangular and square functionally graded plates (FGPs) based on the classical plate theory [4,5]. They used energy method and reached to the closed-form solutions. They have also investigated thermal buckling of FGPs based on the first order displacement field [6-8]. They obtained buckling loads by solving the system of five stability equations. Najafizadeh and Eslami studied the thermoelastic stability of circular functionally graded plates [9,10].Ma and Wang employed the third order shear deformation plate theory to solve the axisymmetric bending and buckling problems of functionally graded circular plates [11]. Three-dimensional thermal buckling analysis of functionally graded materials, using finite element method, is reported by Na and Kim [12].

The purpose of this paper is to develop the two variable refined plate theory for thermal buckling analysis of functionally graded plates. The present theory satisfies equilibrium conditions at the top and bottom faces of the plate without using shear correction factors. Governing equations are derived from the principle of minimum total potential energy. Navier solution is used to obtain the closed-form solutions for simply supported FGM plate. To illustrate the accuracy of the present theory, the obtained results are compared with finite element method and results of the first-order shear deformation theory.

Refined plate theory for FG plates - Governing equations: The displacement field of a rectangular FGM plate, based on the two variable refined plate theory, can be expressed as

$$U(x,y,z) = u(x,y) - z\frac{\partial w_b}{\partial x} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \tag{4}$$

$$V(x,y,z) = v(x,y) - z\frac{\partial w_b}{\partial y} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y}$$

$$W(x,y,z) = w_b(x,y) + w_s(x,y)$$

Constitutive relations: Consider a FGM plate made of ceramic and metal, the material properties of FGM such as material properties vary continuously across the thickness according to the following equations, which are the same as the equations proposed by Reddy et al. and Praveen et al. [3,4]:

$$E(z) = E_m + E_{cm} V_f(z) \qquad E_{cm} = E_c - E_m \tag{7}$$

$$\alpha(z) = \alpha_m + \alpha_{cm} V_f(z) \qquad \alpha_{cm} = \alpha_c - \alpha_m$$

$$\nu(z) = \nu_0$$

Buckling analysis of FGM plates: The critical buckling temperature or temperature differences ΔT_{cr} with respect to the material gradient indexes of the plate are calculated for functionally graded squares plates with different relative thickness of the plate under uniform temperature rise, linear and nonlinear temperature distribution across the thickness using the two variable refined plate theory and are plotted in Figs. 3-5. It is clear that the critical temperature under non linear temperature rise is higher than that under linear temperature rise, which is higher than that under uniform temperature rise, whatever the gradient index k is. In all type of temperature cases (uniform, linear, nonlinear), the critical temperature change decreases, when the volume fraction index k a/h is increased. On the other hand, the critical temperature change increases, when the relative thickness of the plate a/h is decreased.

Conclusions: The thermal buckling analysis of functionally graded plates is carried out by using two variable refined plate theory. Material properties varied continuously in the thickness direction according to a simple power law distribution in terms of the volume fraction of a ceramic and metal. Numerical results of a simply isotropic square and rectangular plate under uniform and linear temperature rise compare well with those of the previous works. Further, the thermal buckling behavior under uniform, linear and nonlinear temperature rise across the thickness is analyzed for simply Al/Al2O3 FGM plates. The critical temperature decreases as volume fraction index is increased. In addition, the critical temperature increases when the geometric parameter b/h is decreased. The critical temperature under nonlinear temperature rise is higher than that under linear temperature rise and uniform temperature rise.

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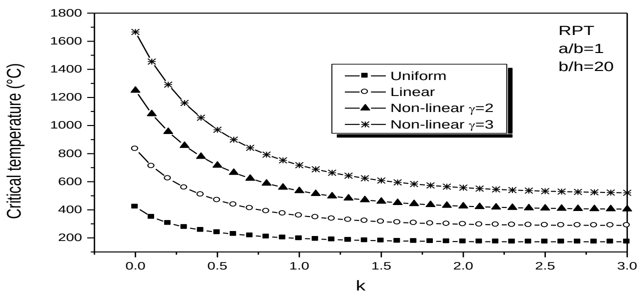

Figure 3. Critical buckling temperature of FGM plate under uniform linear and nonlinear temperature rise vs gradient index of the plate.